# MOTION OF LIQUID DROPS IN ONE-DIMENSIONAL GAS FLOW

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Formulas for calculating the velocity of liquid drops entrained by a gas flow are obtained. These formulas can be used to analyze the effect of dimensionless similarity criteria ($\tau_g$, Re, We, Kn, etc.) on the velocity of the drops.

In deriving the equations of particle motion we assume that 1) electrostatic and gravitational forces acting on the particle are negligibly small in comparison with the force due to the gas flow; 2) the particles do not collide with one another; 3) mass and heat transfer between the phases has no appreciable effect on the motion of the particles. On the adoption of these assumptions the equation of motion of an individual particle of arbitrary shape entrained by a one-dimensional gas flow can be written in the following form:

$$R_1\frac{dc_l}{d\tau_l}=\frac{3k_M\psi(1+k_y)}{L\bar{\rho}}(c_v-c_l)^2+R_2\frac{dc_v}{d\tau_v}. \quad (1)$$

With account for the relations $d\tau_v = dz/c_v$ and $d\tau_l = dz/c_l$, Eq. (1) takes the dimensionless form

$$R_1\frac{d\bar{c}_l}{d\bar{z}}=\frac{\bar{c}_v-\bar{c}_l}{\tau_g\bar{c}_l}+R_2\frac{\bar{c}_v}{\bar{c}_l}\frac{d\bar{c}_v}{d\bar{z}}. \quad (2)$$

Introducing the slip factor into Eq. (2) we obtain the criterial equation of motion of the particle in its final form:

$$\frac{d\nu}{d\varphi}=\frac{(\nu-\nu_{v1})(\nu_{v2}-\nu)}{\nu}, \quad (3)$$

where

$$\nu_{v1}=2(1+R_2\bar{\tau}_g)/(1+\sqrt{\Delta}); \quad (4)$$

$$\nu_{v2}=(1+\sqrt{\Delta})/(-2R_1\bar{\tau}_g); \quad (5)$$

$$\varphi=\ln\bar{c}_v;\ \Delta=1+4R_1\bar{\tau}_g(1+R_2\bar{\tau}_g).$$

An analysis of Eq. (3) shows that, in the case of monotonic variation of $\bar{c}_v$ in function $\bar{z}$, $\nu_{vi}$ ($\nu_{v1}$ or $\nu_{v2}$) is the limiting value of the slip factor $\nu$, which it assumes after completion of the relaxation of momentum transfer between the particle and the gas.

With increase in $\bar{z}$ the value of the slip factor $\nu$ varies from the initial value $\nu_0$ to $\nu_{v1}$ (if $\bar{\tau}_g$ is in the range from $+\infty$ to $t_p$) or from $\nu_0$ to $\nu_{v2}$ (in the case of $t_r \geq \bar{\tau}_g > -\infty$). Here

$$t_p=-[1/(2R_1p)];\quad p=1+\sqrt{1-(R_2/R_1)}, \quad (6)$$

$$t_r=-[1/(2R_1r)];\quad r=1-\sqrt{1-(R_2/R_1)}. \quad (7)$$

In the interval $t_r < \bar{\tau}_g < t_p$ the limiting value of $\nu$ cannot be attained physically owing to the finiteness of the distance $\Delta z$. If $\nu_0 = \nu_{v1}$, then the factor $\nu$ over the whole path of motion of the particle is constant in magnitude and equal to $\nu_0$. If $\nu_0 \neq \nu_{v1}$ the slip factor asymptotically approaches $\nu_{vi}$ as $\bar{z}$ increases. Depending on the ratio $\nu_0/\nu_{vi}$ the slip factor can either decrease or increase with increase in $\bar{z}$. From Eq. (3) we obtain the following relationships for some self-similar cases of motion of particles in a flow.

a) If $\bar{\tau}_g \to \infty$, and $R_1$ and $R_2$ are independent of $\varphi$ and $\nu$, then

$$\nu=\frac{1}{\bar{c}_v}\sqrt{\nu_0^2+\frac{R_2}{R_1}(\bar{c}_v-1)(\bar{c}_v+1)}. \quad (8)$$

When $S_{vis} = 0$ relationship (8) describes the motion of particles in an ideal fluid.

b) If $\bar{\tau}_g \to \infty$ and $R_2 = 0$, then from (8) we have

$$\nu=\nu_0/\bar{c}_v \text{ for } c_l=(c_l)_0. \quad (9)$$

In this case the vapor and liquid phases move independently of one another without momentum transfer to one another. This motion of particles in a viscous gas can be called "minimum-equilibrium" motion.

c) It follows from the relaxation equation (3) that $\Phi = |\nu - \nu_{vi}|$ characterizes the degree of nonequilibrium of the momentum transfer between the phases. The sign of $\nu - \nu_{vi}$ indicates the direction of the transfer process. If $\Phi$ = const, the motion of the two-phase medium proceeds with a constant (kinematic) degree of nonequilibrium. In the special case in which $\Phi = 0$ the motion of the two-phase medium is of an equilibrium nature (in the sense of absence of relaxation of the slip factor $\nu$). If $\Phi = 0$, then

$$\nu=\nu_0=\nu_{vi}=\text{const}.$$

d) If $\Phi_0 = \Phi = 0$ and $\tau_g = 0$, then if $d\bar{c}_v/d\bar{z}$ is finite, $\nu = 1$. In this case the motion of particles in a viscous gas is "maximum-equilibrium" motion, since there is complete equalization of the velocities of the phases. If $\Phi_0 \neq 0$, but $\tau_g = 0$, the motion of particles in the viscous phase will be of the maximum-equilibrium type only when $\bar{z} > 0$. In this case the function $\nu(\bar{z})$ has a discontinuity at the origin of coordinates. The size of the discontinuity in $\nu$ is $\Delta\nu = |1 - \nu_0|$. For any boundary conditions the equalization of the velocities of the phases in the last case terminates at the very start of the $\bar{z}$ axis.

e) The factor $\nu$ tends to unity in two special cases: 1) when the particles move in a gradient-free gas flow ($d\bar{c}_v/d\bar{z} = 0$) and 2) in the case of maximum-equilibrium motion of the two-phase medium ($\tau_g = 0$). In both cases $\bar{\tau}_g = \tau_g(d\bar{c}_v/d\bar{z}) = 0$ and, as follows from formula (4), $\nu_{vi} = 1$.

If $R_1$, $R_2$ and $\bar{\tau}_g$ are independent of $\varphi$ and $\nu$, then Eq. (3) is converted to an equation with separable variables and is easily integrated. The result of integration is given below.

1) If $t_r > \bar{\tau}_g > t_p$ (the whole region of confusor, gradient-free, and partially diffusor flow of vapor), then

$$\bar{c}_v = \left[\frac{\nu_0 - \nu_{v1}}{\nu - \nu_{v1}}\right]^{\left(-\frac{\nu_{v1}}{\nu_{v1}-\nu_{v2}}\right)}\left[\frac{\nu_0 - \nu_{v2}}{\nu - \nu_{v2}}\right]^{\left(-\frac{\nu_{v2}}{\nu_{v2}-\nu_{v1}}\right)} \tag{10}$$

2) If $\bar{\tau}_g = t_r$ or $\bar{\tau}_g = t_p$ (diffusor flow of vapor), then

$$\bar{c}_v = \frac{\nu_{vi} - \nu_0}{\nu_{vi} - \nu} \exp\left\{-\frac{\nu_{vi}(\nu - \nu_0)}{(\nu_{vi} - \nu_0)(\nu_{vi} - \nu)}\right\}. \tag{11}$$

When $\bar{\tau}_g = t_r$, $\nu_{vi} = r$, and when $\bar{\tau}_g = t_p$, $\nu_{vi} = p$.

3) If $t_r < \bar{\tau}_g < t_p$ (diffusor flow of vapor), then

$$\bar{c}_v = \sqrt{\frac{\nu_0^2 R_1 \bar{\tau}_g + \nu_0 - (1 + R_2\bar{\tau}_g)}{\nu^2 R_1 \bar{\tau}_g + \nu - (1 + R_2 \bar{\tau}_g)}} \times$$
$$\times \exp\left\{\frac{1}{\sqrt{-\Delta}}\left[\operatorname{arctg}\frac{2R_1\bar{\tau}_g\nu + 1}{\sqrt{-\Delta}} - \operatorname{arctg}\frac{2R_1\bar{\tau}_g\nu_0 + 1}{\sqrt{-\Delta}}\right]\right\}. \tag{12}$$

4) For a gradient-free flow of gas Eq. (10) is converted to the following form:

$$\bar{z} = R_1\tau_g\left[\nu_0 - \nu + \ln\frac{\nu_0 - 1}{\nu - 1}\right]. \tag{13}$$

In the more general cases of motion of particles in gas flows Eq. (3) can be solved by one of the methods of approximate integration. However, if high accuracy is not required a first approximation of $\nu$ can be obtained in the following way. The interval T within which the function $\bar{c}_v$ is determined is divided into several regions of length $\Delta z$. It is assumed that within each region $\Delta z$ the velocity $\bar{c}_v$ varies according to a linear law, that $\tau_g$, $R_1$, and $R_2$ are constants and equal to the mean values in this interval:

$$\bar{c}_v = 1 + (\varphi_v - 1)\bar{z}, \tag{14}$$

where

$$\varphi_v = (c_v)_1/(c_v)_0; \quad \bar{\tau}_g = \tau_g(\varphi_v - 1).$$

The factor $\nu$ in the considered case can be calculated from formulas (10)–(13).

As an example in Fig. 1 we have plotted $\nu$ against $\bar{z}$ and $\tau_g$ for two boundary conditions: $\nu_0 = 0$ and $\nu_0 = 2$ with $R_1 = 1$ and $\varphi_v = 2$. The figure shows the range of variation of $R_2$ within which the effect of "mass addition" on $\nu$ does not exceed 1%. Figure 1 shows that

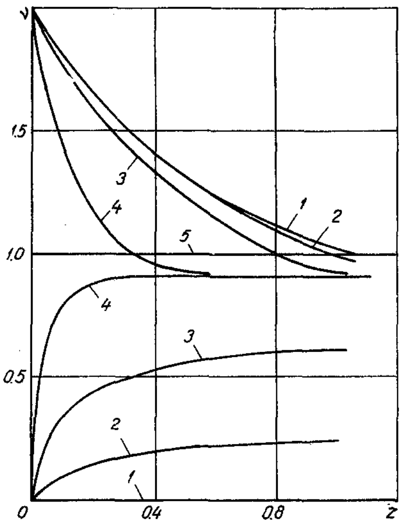

Fig. 1. Slip factor $\nu$ as a function of coordinate $\bar{z}$, and parameters $\tau_g$ and $\nu_0$ for $\varphi_v = 2$ and $R_1 = 1$: 1) $\tau_g \geq 10^3$, $R_2 = 0$; 2) $\tau_g = 10$, $R_2 \leq 10^{-3}$; 3) $\tau_g = 1$, $R_2 \leq 10^{-3}$; $\tau_g = 1$, $R_2 \leq 10^{-3}$; 4) $\tau_g = \leq 0.1$.

when $0 \leq \nu_0 \leq 2$ the region of "relaxation of motion" of the drop is practically confined to the range $10^{-3} < \tau_g < 10^3$.

Formulas (3)–(13) can be used for practical calculation of the factor $\nu$ only in the case in which $R_1$, $R_2$ and $\tau_g$ are determined. For engineering calculation of $\tau_g$ it is more convenient to use the second form of $\tau_g$ given in the notation.

The factors $k_D$, $k_\beta$, and $k_g$ can be calculated by means of the following relationships given in [1] for the case of solid nondeformable particles:

$$k_D = \left(1 - 0.6\frac{D}{D_c}\right)\left(1 - \frac{D}{D_c}\right)^2 \tag{15}$$

for laminar flow of the gas round the particle;

$$k_D = \left[1 + 2.1\frac{D}{D_c}\left(1 - \frac{D}{D_c}\right)\right]\left(1 - \frac{D}{D_c}\right)^2 \tag{16}$$

for turbulent flow round the particle, where $D_c$ is the hydraulic diameter of the cross section of the channel;

$$k_\beta = (1 - \beta)^n; \quad n = 2.25 - 4.5. \tag{17}$$

With sufficient accuracy we can assume that when $D_c/D \geq 10$, $k_D = 1$, and when $\beta \leq 0.01$, $k_\beta = 1$.

The data for calculation of $k_g$ in relation to Re and coefficient $k_G$ are given in the table.

If $\psi''$ is calculated from Klyachko's formula [2], then for $3 < \mathrm{Re} < 400$ we obtain

$$k_{sph} = 1 + \frac{1}{6}\mathrm{Re}^{2/3}; \tag{18}$$

for $\mathrm{Re} \leq 1$

$$k_{sph} = 1 + \frac{3}{16}\mathrm{Re} - \frac{19}{1280}(\mathrm{Re})^2. \tag{19}$$

From the data of [2]

$$k_p = 1 + 2B_1\,\mathrm{Kn} + 2B_2\,\mathrm{Kn}\exp\left(-\frac{B_3}{2\mathrm{Kn}}\right), \qquad (20)$$

where

$$B_1 = 0.7-0.9; \quad B_2 = 0.2-0.3; \quad B_3 = 1-3.$$

The movement of liquid particles entrained by a flow of gas, as distinct from the movement of nondeformable particles, is accompanied by several additional processes, one of which is "pliability" of the shape of the particle. If we use the relationship We = = We($\gamma$) obtained by Klyachko [3] for the case of a drop deformed into an oblate spheroid, we can obtain the required relationship $k_G = k_G(\mathrm{We})$ in the form given in Fig. 2.

The relationship in Fig. 2 provides a means of determining the nonsphericity factor $k_G$ for given conditions of flow (We, Re) round the drop. From the known Re and $k_G$ we use the formulas in the table to calculate the dynamic form factor $k_g$, which thus takes into account the effect of pliability of the shape of the liquid particle on the drag coefficient $\psi$.

In the majority of cases encountered in practice $1 \gg s/\bar{\rho}$. On this basis we can assume $R_1 = 1$. If the particle has a simple geometric shape then s is determined from the corresponding tables, which are well known in the literature. (For a sphere, for instance, $s_i = 0.5$; for a cylinder lying across the flow $s_i = 1.0$, and so on). The calculation of $s_{vis}$ and $k_a$ entails considerable difficulties. Formulas suitable for practical calculation have been obtained only for the simplest forms of motion. For instance, in the case of motion of a spherical particle in a gas oscillating harmonically with angular frequency $\omega$, according to the data of [2], we have

$$k_a = \frac{3}{2}\sqrt{\frac{\omega\tau_l}{\bar{\rho}}}; \quad s_{vis} = \frac{k_a}{\omega\tau_l}.$$

The author of [2] thinks that if Re does not exceed a few hundred, the factor $k_a$ can be neglected in the calculation.

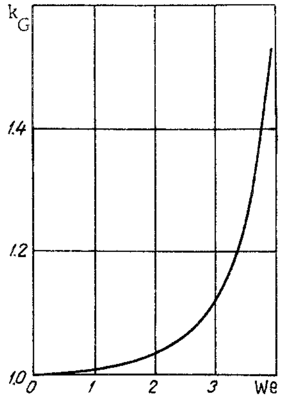


Fig. 2. Form factor $k_G$ as a function of We for initially spherical drop of liquid.

As was noted above, the slip factor $\nu$ is calculated from the average value of $\tau_g$ in the interval $\Delta z$. However, at the beginning of the calculation the velocity of the liquid at $\bar{z} = 1$ is unknown (and, hence, the average value of $\tau_g$ in the interval $\Delta z$ is unknown). For this reason $(c_l)_1$ and $\tau_g$ are calculated by the method of successive approximations.

NOTATION

c is the velocity; $\bar{c} = c/(c_V)_0$ is the dimensionless velocity; $(c)_1$ is the velocity at end of region $\Delta z$; D is the diameter of the spherical drop, equal to characteristic dimension L of particle of arbitrary shape; $k_g = (\Omega/\Omega_{sph})_{V=idem}$ is the geometric form factor of particle; $k_D$ is the particle-motion restraint factor, which allows for effect of channel walls on $\psi$; $k_g = (\psi'/\psi_{sph})_{Re=idem,\ Kn=idem}$ is the dynamic form factor of particle; $k_M = \Omega_M/\Omega$ is the particle area factor (for sphere $k_M = 0.25$); $k_p = (\psi''/\psi_{sph})_{D=idem}$ is the factor which depends on Knudsen number and allows for "slipping effect" in boundary layer of particle; $k_{sph} = (\psi''/\psi_s)$ is the factor allowing for additional effect of Re on drag coefficient of spherical particle ($\psi''$) at Re > 0.1; $k_a$ is the factor allowing for effect of acceleration on dissipative force; $k_\beta$ is the particle motion restraint

Relationships for Calculation of $k_g$ in Relation to $k_G$ and Re

| $k_G$ | Re | $k_g$ |
|---|---|---|
| 1 ÷ 1.5 | $2\cdot10^3 < \mathrm{Re} < 2\cdot10^5$ | $k_g \simeq 12.4 - \frac{11.4}{k_G}$ |
| 1.15 ÷ 1.2 | 30 ÷ 400 | $\frac{\mathrm{Re}^{0.8}}{6.78 + 1.13\cdot\mathrm{Re}^{2/3}}$ |
| 1.4 ÷ 1.5 | 45 ÷ 300 | $\frac{\mathrm{Re}^{0.8}}{4 + 0.667\cdot\mathrm{Re}^{2/3}}$ |
| 1.15 ÷ 1.2 | 1 ÷ 30 | $\frac{\mathrm{Re}^{0.5}}{2.45 + 0.409\cdot\mathrm{Re}^{2/3}}$ |
| 1.4 ÷ 1.5 | 1 ÷ 45 | $\frac{\mathrm{Re}^{0.5}}{1.26 + 0.21\cdot\mathrm{Re}^{2/3}}$ |
| 1.0 ÷ 1.5 | < 1 | $\frac{1}{k_{sph}}$ |

factor allowing for effect of presence of neighboring particles on $\psi$; $(k_D k_\beta)^2 = \psi'/\psi$; $Kn = l/L$ is the Knudsen number; $L = 6V/\Omega$ is the characteristic dimension of particle of arbitrary form; $l$ is the mean free path of molecule of gaseous phase; $R_1 = 1 + (s/\bar{\rho})$ and $R_2 = (1 + s)/\bar{\rho}$ are factors allowing for "mass-addition" effect; $Re = [|c_v - c_l|\rho_v]/\mu_l$ is the Reynolds number; $s = s_i + s_{vis}$ is the total mass addition factor; $s_i$ is the mass addition factor in case of motion of particle in ideal fluid; $s_{vis}$ is the mass addition factor making additional allowance for effect of viscosity of gas; V is the volume of particle; $We = [D\rho_v(c_v - c_l)^2]/\sigma$ is the Weber number; z is the linear coordinate; $\Delta z$ is the distance traversed by particle; $\bar{z} = z/\Delta z$ is the dimensionless coordinate; $\beta$ is the instantaneous volume concentration of liquid phase; $\gamma$ is the ratio of major semiaxis of oblate spheroid to ; minor semiaxis; $\mu$ is the dynamic viscosity; $\nu = c_l/c_v$ is the slip factor; $\rho$ is the density; $\bar{\rho} = \rho_l/\rho_v$ is the density ratio; $\sigma$ is the surface tension of liquid; $\tau$ is the time; $\bar{\tau}_g = \tau_g(d\bar{c}_v/d\bar{z})$ is the generalized similarity criterion of motion of particle of arbitrary shape; $\tau_g$ is the dimensionless structure-time criterion of similarity of motion of particle of arbitrary shape (generalized Stokes criterion) $\tau_g = L\bar{\rho}(c_v)_0/3k_M\psi(1 + k_a)\Delta z|c_v - c_l|$ or $\tau_g = \bar{\tau}_l[k_p(k_D k_\beta)^2/4k_M k_{sph} k_g(1 + k_a)]$; $\bar{\tau}_L = \bar{\tau}_L = [\tau_l(c_v)_0]/\Delta z$ is the dimensionless structure time criterion of similarity of motion of spherical particle in case of Stokes motion (Stokes number); $\tau_l = [\rho_l D^2]/[18\mu_v]$ is the time of "relaxation of motion" of sphere in case of Stokes motion; $\psi$ is the drag coefficient of particle (variation of dissipative force with acceleration neglected); $\psi'$ is the drag coefficient of single particle of arbitrary shape in unconfined space; $\psi_{sph}$ is the drag coefficient of spherical particle at arbitrary Re and Kn; $\psi''$ is the drag coefficient of sphere at arbitrary Re; $\psi_s = 24/Re$ is the drag coefficient of sphere in case of Stokes motion; $\Omega$ is the surface area of particle; $\Omega_M$ is the area of middle section of particle; $\Omega_{sph}$ is the surface area of spherical particle. Subscripts: $l$ denotes liquid (solid) phase; 0 denotes parameter at $\bar{z} = 0$; v denotes vapor (gaseous) phase.

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